# Extended control charts

The control chart types listed below are recommended as alternative and additional tools to the Shewhart control charts. When compared with classical charts, they have some advantages and some disadvantages. More difficult construction and interpretation are on the minus side, while much larger sensitivity (up to an order of magnitude) to the shifts of the mean value (CUSUM), applicability even for data showing long cycles, trend or nonconstant variance (EWMA – dynamic modification) are on the plus side. When several correlated variables are controlled simultaneously, the Hotelling control chart is appropriate.

## *CUSUM*

| Menu: | QC.Expert | Other Control Charts | CUSUM |
|-------|-----------|----------------------|-------|

CUSUM control chart and its efficient modification by Lucas (V-mask is not needed) is recommended mainly when a process mean shift needs to be detected fast. The method is based on summing differences from the goal cumulatively (Cumulative SUMs). Imbalance of differences from the goal (more differences of one sign) is quickly detected. Sensitivity of this chart is given by the *k* parameter, which can be inputted in the CUSUM chart dialog panel. When the data are normal and independent, the technique is much more efficient than the Shewhart X-bar or X-individual chart – number of data points necessary to detect 1 standard deviation departure from the baseline is about 10 times smaller. When a point falls outside the limits, it is taken as an alarm signal. When remedy steps are taken immediately after obtaining the signal, the FIR technique can be applied. The technique is useful as a quick check of whether the corrective action was successful. It is based on moving the point immediately following the remedy action just below/above the limit on the same side where the previous violation occurred. If the action was not successful, the limit is violated again.

### Data and parameters

The CUSUM chart parameters are inputted in the CUSUM chart dialog panel, see Figure 1. *K* coefficient is entered in the *Sigma value* section. It specifies how large deviation in units of standard deviation from a specified baseline (goal or target) should be detected. Usual values are 1 or 0.5. When the value is too small, false alarms are frequent. On the other hand, when the coefficient value is too large, efficiency of the chart decreases. Limits are specified in the *Limit* field. 5 is recommended for a normal regimen and 4 for a strengthened regimen. Data columns are selected upon clicking the *Select columns* button. When several columns are selected, rows should correspond to different subgroups, since the row means are computed and used for further analysis. When the baseline and standard deviation are known, they can be entered manually upon checking *Manual entry*. FIR is enabled when checking *FIR*, see the previous paragraph.
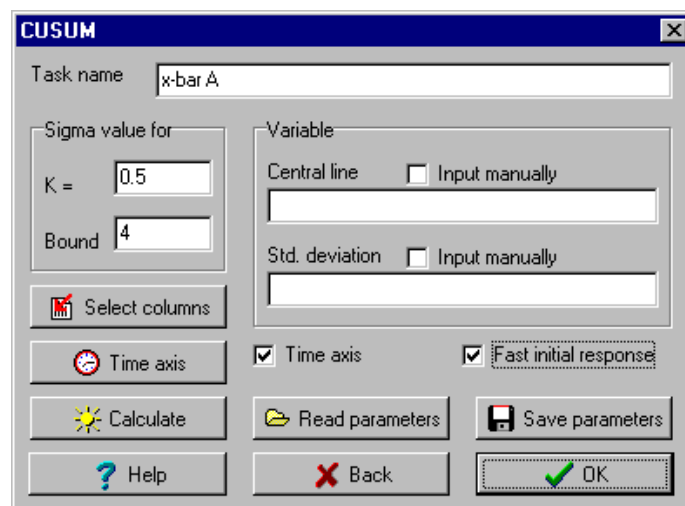


**Fig. 1 CUSUM chart dialog panel**

## Protocol

| | |
|---|---|
| Number of data points | Number of data rows used for chart construction. |
| Baseline | Target value, given or computed from data. |
| Standard deviation | Standard deviation value, given or computed from data. |
| Detectable shift | Minimum shift from the target to be detected. |
| FIR | Yes or No indicating whether the technique was used. |
| Limits +- | Inputted limits. |
| Violating (+) | Number of data points violating upper limit. |
| Violating (-) | Number of data points violating lower limit |

## Graphs
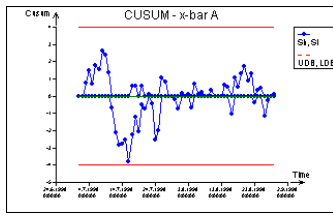
| | |
|---|---|
| CUSUM chart  | CUSUM chart for detecting systematic changes from the target (central line). Violation of the limits (red lines) is taken as an alarm signal. |

## *EWMA*

| Menu: | QC.Expert | Other Control Charts | EWMA |
|---|---|---|---|

The Exponentially Weighted Moving Averages (EWMA) chart is based on the values $X_i = W.x_i + (1 - W).X_{i-1}$. $W$ $(0 < W < 1)$ is a weight specifying how fast is the EWMA response to change in the process mean. When W=1, the chart is equivalent to the X-bar or X-individual chart. Higher W values produce smoother chart which is less sensitive to sudden changes. This module can also produce a dynamical EWMA modification, useful when the controlled process shows long term oscillations or trend so that other chart types cannot be used. Such a situation can be indicated e.g. by significant autocorrelation or time trend , found in the Basic data analysis module. The dynamical EWMA chart tolerates long term process mean or variance changes, but identifies short term, sudden changes. The dynamical EWMA chart is controlled by the A parameter inputted in the dialog panel. The EWMA residual chart is used for the same purpose as the dynamical EWMA chart.

### Data and parameters

Data are organized in one or more columns. When more than one column is selected, row means are computed and used in subsequent analyses, so that rows should correspond to subgroups. Baseline (target mean value) and standard deviation are entered in the EWMA control chart dialog panel., checking *Manual entry*, see Figure 2. Unknown values can be estimated from data. At least one chart type must be selected and W, A weights entered. When Manual entry is not checked, the program computes baseline and limits from data. The Save parameters button saves parameter values to a file. The Read parameters button reads parameters from a specified file. The OK button runs computations, producing protocol and graphical output.
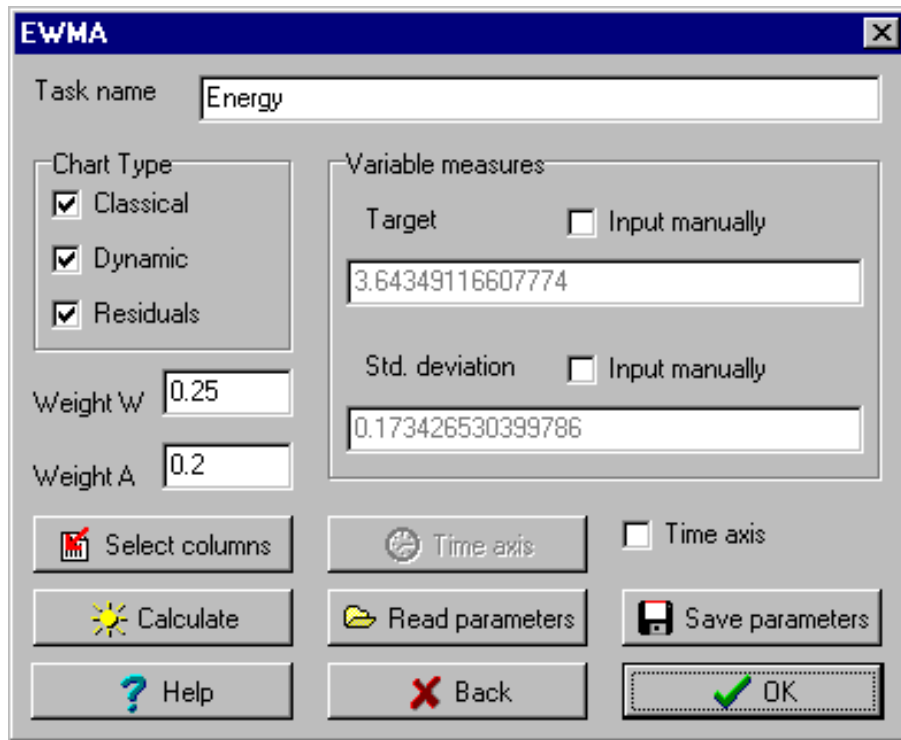
**Fig. 2 EWMA chart dialog panel**

## Protocol

| | |
|---|---|
| Number of data points | Number of data rows |
| Baseline | Baseline (target for mean) inputted in the dialog panel (see Figure 2 )or computed. |
| Standard deviation | Standard deviation ) inputted in the dialog panel (see Figure 2. |
| Weight W for EWMA | Weight for the EWMA construction. |
| Weight A for dynamic EWMA | Weight for dynamic the EWMA construction. |
| Number of data points out of classical control limits | Number of data points violating the EWMA control limits. |
| Number of data points out of dynamical control limits | Number of data points violating dynamical the EWMA control limits. |
| Number of residuals out of $\pm 3\sigma$ limits | Number of data points violating residual the EWMA control limits. |
| Mean square | Mean squared difference from the baseline. |

## Graphs
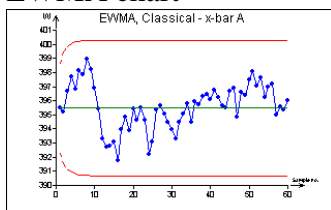
| | |
|---|---|
| EWMA chart  | EWMA chart with nonconstant limits. Violation of the red limits is taken as an alarm signal. |
| Dynamic EWMA chart | Dynamic EWMA chart tolerates slow changes in the mean. |

| | |
|---|---|
|  | Violation of the control limits (red) is taken as an alarm signal. |
| Residual EWMA chart  | Residual EWMA chart has similar use as the dynamic EWMA chart. Violation of the control (red) limits is taken as an alarm signal |
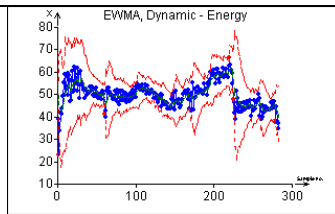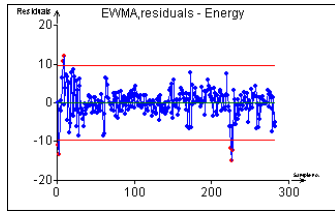
## *Hotelling T2 chart*

| Menu: | QC.Expert | Other Control Charts | Hotelling chart |
|---|---|---|---|

Hotelling control chart is used when several correlated variables are controlled simultaneously. Within this module, the data are assumed to consist of individual (multivariate) data points, hence the chart is a multivariate analogue of the X-individual chart. Such data might be used in X-individual control charts for separate variables only if the variables are not correlated – their correlation coefficients should not be significant, see the Correlation module description. Hotelling chart reduces the information about individual variables to one characteristic, namely distance from the (multivariate) mean computed with respect to the covariance matrix (Mahalanobis distance). The distance cannot be negative and respects interrelationships among different variables. The lower limit is always set to zero.

## Data and parameters

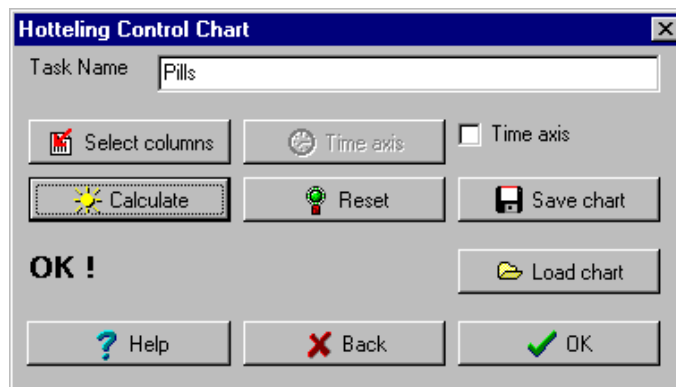Data are organized in columns, corresponding to different variables. Rows with missing data are ignored completely.



**Fig. 3 Hotelling chart dialog panel**

*Setting chart parameters*

Parameters are estimated from a training dataset, obtained when the process was under control. Upon clicking the *Select columns* button, data columns are specified. The *Reset* button resets all parameter values set previously. The *Calculate* button computes mean vector and covariance matrix. Parameters can be saved to a file by *Save parameters* button. The OK button starts the chart computations. The baseline is given by the mean vector estimated from the training dataset. Allowable amount of variability is given by complete covariance matrix, estimated from the training dataset. The mean vector is outputted to the protocol. The covariance matrix can be inspected by viewer called from the *QC.Expert-Viewer-Hotelling control chart* menu.

*Control chart application*

Chart parameters obtained previously and saved in a file are read by clicking the *Read parameters* button. The OK button runs computations.

## Protocol

| | |
|---|---|
| Column name | Data column names. |
| Target mean | Means computed from the training dataset. |
| Mean | Means for the process data for which the chart is applied. |
| **Chart parameters** | |
| LCL, UCL | Lower and upper control limits, lower limit is always set to zero. |
| **Out of limit** | List of data points exceeding the upper limit. |
| Time | Times at which UL was exceeded. |
| Number | Identification of data points exceeding UL. |
| Value | $T^2$ values for data points exceeding UL. |

## Graphs
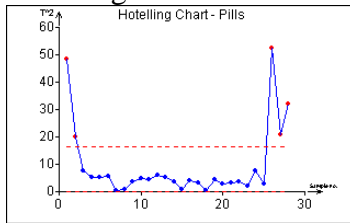
| Hotelling chart | Distances of individual (multivariate) data points from the target mean, expressed as Hotelling $T^2$ values. The other of the two charts shows T values, i.e. square roots of $T^2$. Both charts have exactly the same interpretation. This characteristic takes into account simultaneous behavior of all variables. The lower control limit is always set to zero, the upper control limit is given by a chi-square distribution quantile, depending on the number of variables used. |
|---|---|
|  | |
|  | |

# Multiple control charts

| Menu: | QCExpert | Multiple Charts |
|---|---|---|
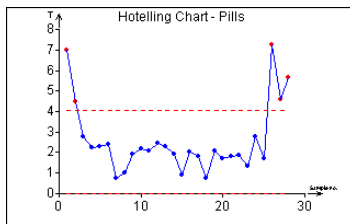
This module is used for quick diagnostics of univariate processes simultaneously by 3 different control charts. It enables the user to assess stability, process changes and outliers and to select a suitable chart type for possible further analysis of similar processes. The module offers Shewhart x-individual chart (without special causes diagnostics), autoregression (AR) control chart and dynamic one-step-ahead prediction EWMA control chart. This module is a simplified alternative to standard control charts.

## *Data and parameters*

Data are expected in columns in the data sheet. Lengths on the data columns may be different. In the *Columns* field in the dialog window (Fig. 4) select all column that are to be analyzed. Check the required chart types (any combination from *Shewhart*, *Autoregression* and *EWMA*, at least one type must be selected) and specify their parameters: *No of data for control limits*, *Order of AR* and *Exponential weight*.
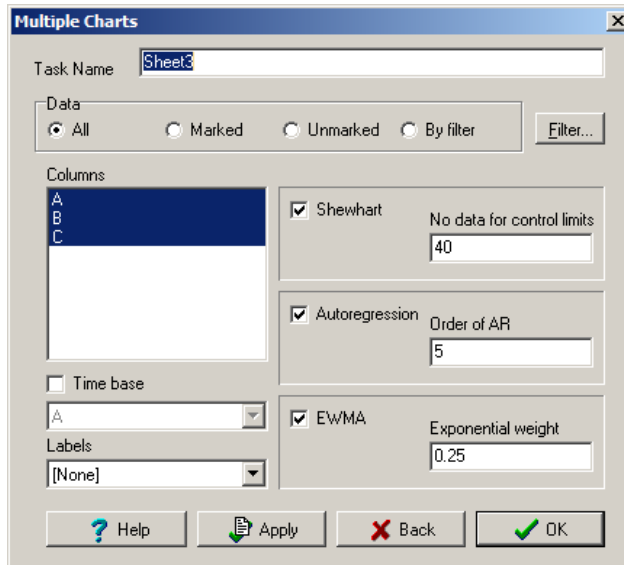
**Fig. 4 Multiple Control Charts dialog window**

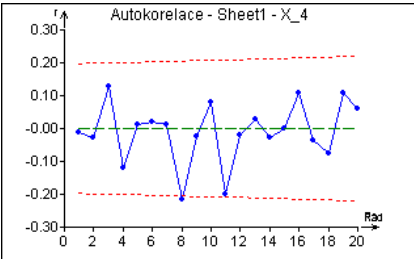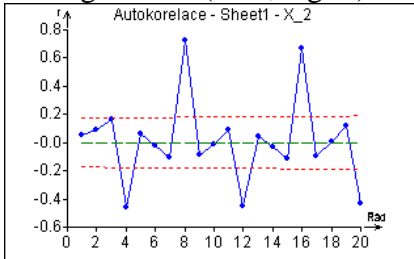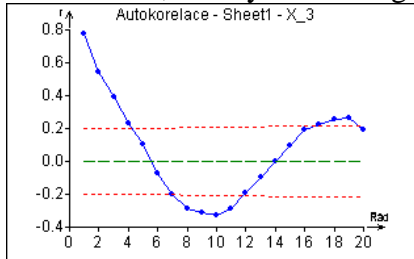*Chart's parameters description –*

*Shewhart: No of data for control limits* – Number $N_{Sh}$ of first values in a column that will be used to calculate control limits (*UCL, LCL*) and the central line (*CL*).

*Autoregression: Order of AR* – Order P of the used autoregression model AR(*P*). The autoregression model coefficients are computed from all data in the respective column. Number of data rows must be at least *P*+2. Usual value is 2 to 10. If there is a striking significant maximum on the autocorrelation function (see the middle column on the illustration below) then the recommended value of AR order is the lag (x-value) at the first maximum.

*EWMA: Exponential weight* – The required value of the exponential weight *W*. Typical values for *W* are between 0.05 and 0.3.

The *Time base* checkbox allows to specify a time axis column which is then common for all columns and all plotted charts. If a column *Labels* is selected the values or test in this column is used to identify selected points in the plotted charts (as in Fig. 5).

The following table is an informative guide for selecting an appropriate chart based on an autocorrelation function (ACF) available from the *Basic Statistics* module. In each column of the illustration there is a specific pattern of data possibly suitable for Shewhart, Autoregression, or EWMA control charts.

| | Autocorrelation function | |
|---|---|---|
| Insignificant, or slightly significant autocorrelation coefficient | Some autocorrelation coefficients are highly significant (here, lag=8) | Significant first autocorrelation coefficient, slowly decreasing |
|  |  |  |
| | **The data** | |
| Independent randomly distributed, no trend | Regular pattern (better visible in | Slow, irregular oscillations and/or linear trend |

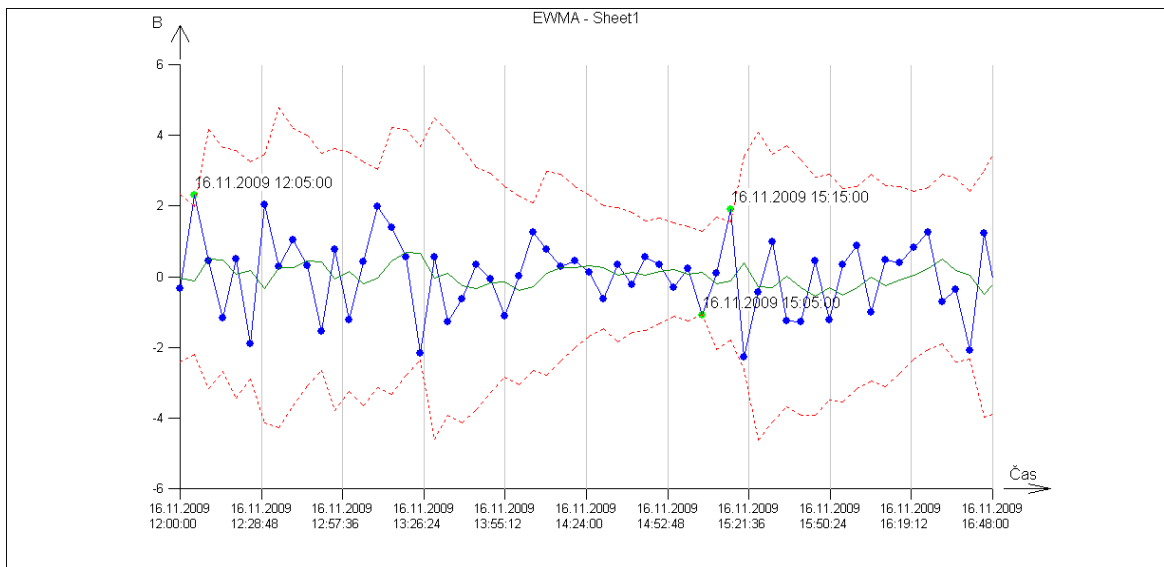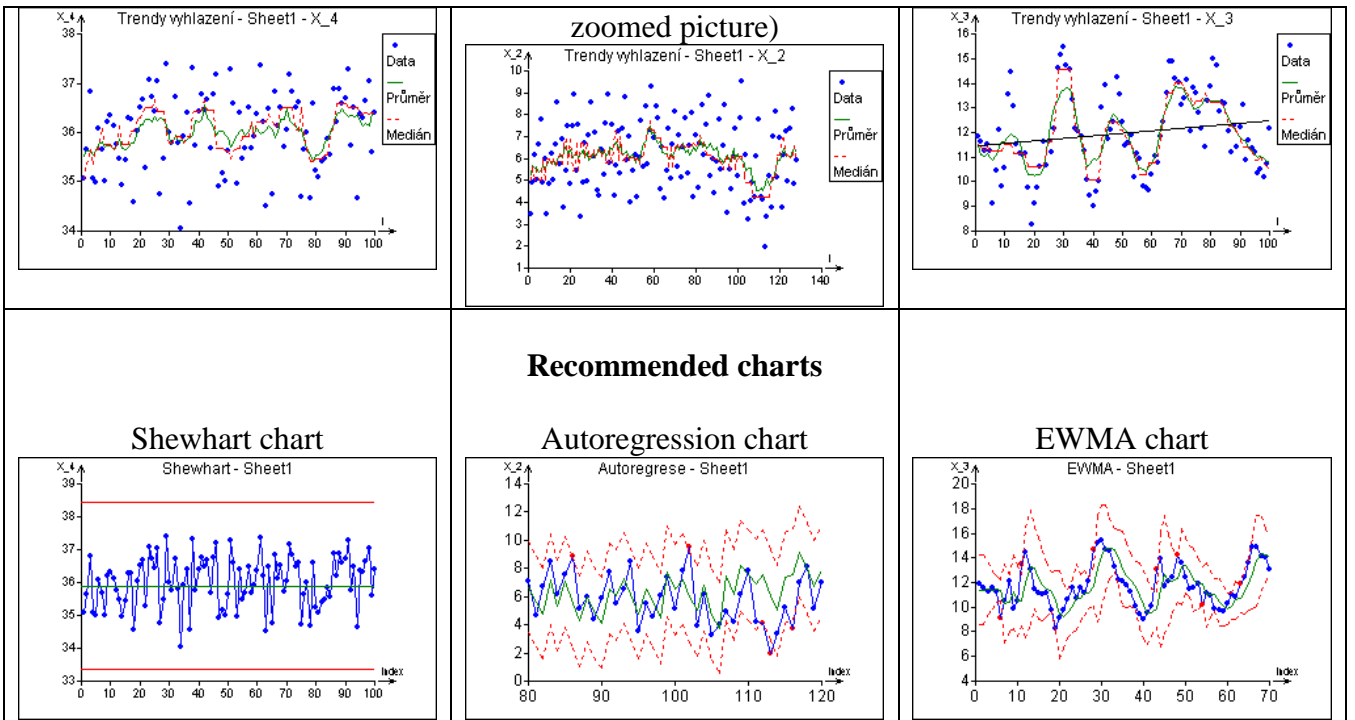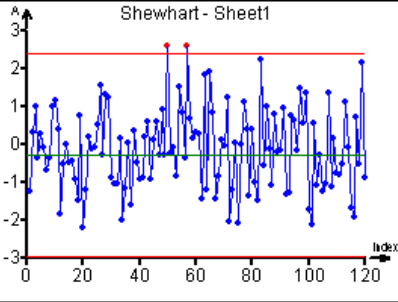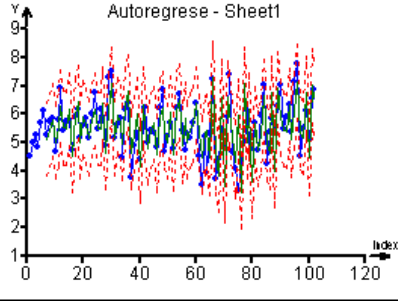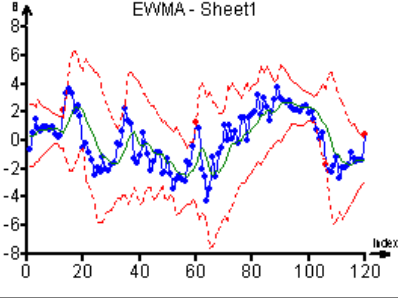**Fig. 5 Identification of selected points in the interactive plot**

## *Protocol*

No text output is generated in this module.

## *Graphs*

| | |
|---|---|
|  | Shewhart control chart. In case of stable, normally distributed data, 99,73% of the points should lie within the red ±3sigma control limits. Excessing control limits is understood as violation of the process stability. |
|  | Autoregression control chart for autocorrelated processes with expected mean (central line, green) and symmetric red ±3sigma control limits. The chart takes into account regular pattern (perhaps caused by mixing output from several production lines or stations). Ret dots lying outside the control limits possibly violate the regular structure of the process data. |
|  | Dynamic EWMA control chart takes into account sloe movement of the process mean value, unexpected fast changes in mean or variance are indicated as process instability. |