

Shewhart control charts

Menu: QC.Expert Control charts

Control charts are constructed to decide whether a process is **under statistical control** and to monitor any departures from this state. This means that stability of some process properties over time is tested using certain statistical assumptions about the process (data it produces). Commonly considered properties are mean, variance (standard deviation), distribution shape or proportion of nonconforming items. The Shewhart control charts were invented in 1932. They are based on monitoring events, which are very unlikely when the controlled process is stable. Any incidence of such an event is taken as an alarm signal suggesting that stability of the process was broken and the process changed. Upon receiving such signal, possible causes of the change should be investigated and some correcting steps taken. One example of such an unlikely event is the situation when the control limits (UCL or LCL) are exceeded. They are constructed as $\pm 3\sigma$ limits, so that when the process is under control, they are exceeded with relative frequency of 0.27%. In addition to the LCL and UCL limits, warning limits (LWL and UWL) are constructed at $\pm 2\sigma$ as well as limits at $\pm\sigma$. More complicated rules specifying event having low probability when the process is under control can be constructed. Some of these rules can be selected in the Shewhart control chart dialog panel, by default all available rules are selected. They are:

1. One point exceeds LCL/UCL.
2. Nine points above/below the central line.
3. Six consecutive points show increasing/decreasing trend.
4. Difference of consecutive values alternates in sign for fourteen points.
5. Two out of three points exceed LWL or UWL limits.
6. Four out of five points are above/below the central line and exceed $\pm\sigma$ limit.
7. Fifteen points are within $\pm\sigma$ limits.
8. Eight consecutive values are beyond $\pm\sigma$ limits.

Shewhart charts are used in two steps:

1. Chart construction
2. Chart application

The goal of the construction step is to specify the central line (CL) and control limits so that they describe the real process correctly. When these values are not given in advance, they are set to mean and interval which contains 99.3% of available training data. For normal data, the interval is constructed $\pm 3\sigma$ interval around arithmetic average, with the statistical characteristics based on observations of the process under control, excluding outliers or otherwise suspect data points. The chart is then applied to control the process, using a specified set of rules.

The QC.Expert offers seven common types of Shewhart control charts:

X-bar and S, X-bar and R, X-individual for continuous variables, np, p, u, c for discrete quality attributes. Continuous data can be transformed to improve normality. Control limits for back transformed data can be asymmetrical.

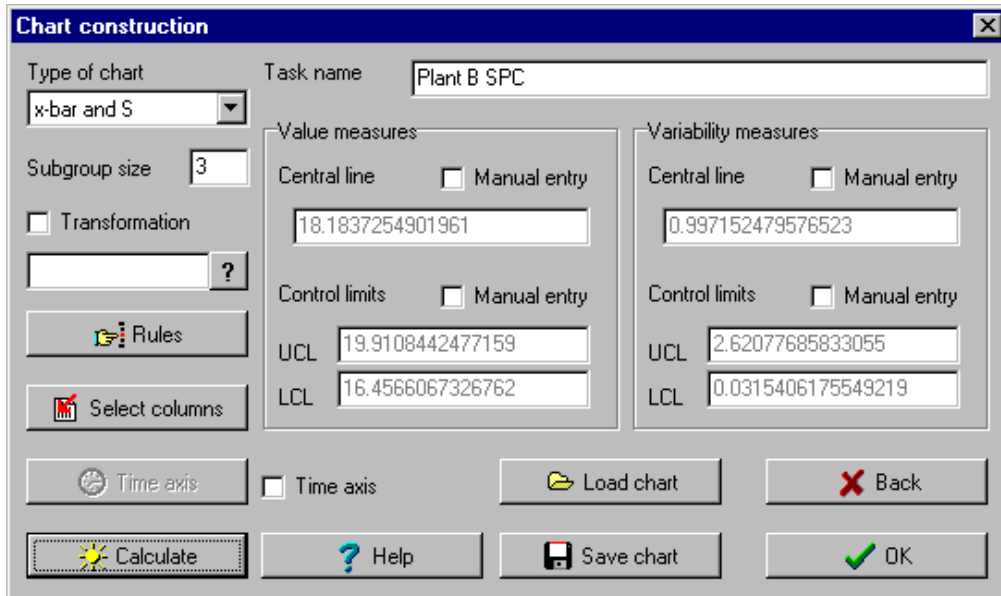


Fig. 1 Shewhart control chart construction dialog panel

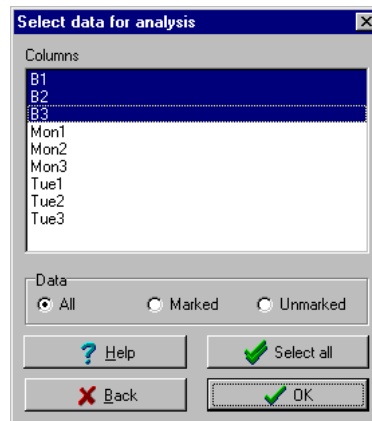


Fig. 2 Rows in selected columns represent subgroups for chart construction

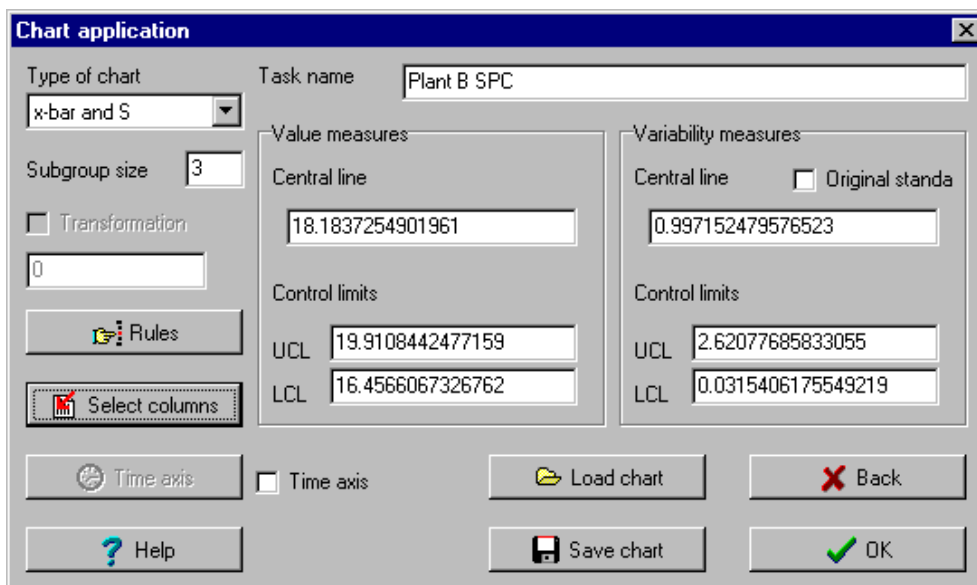


Fig. 3 Shewhart control chart application dialog panel

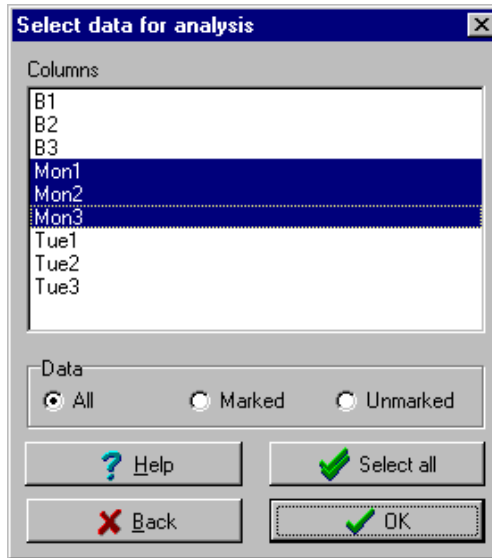


Fig. 4 Columns selected for application of the previously constructed chart

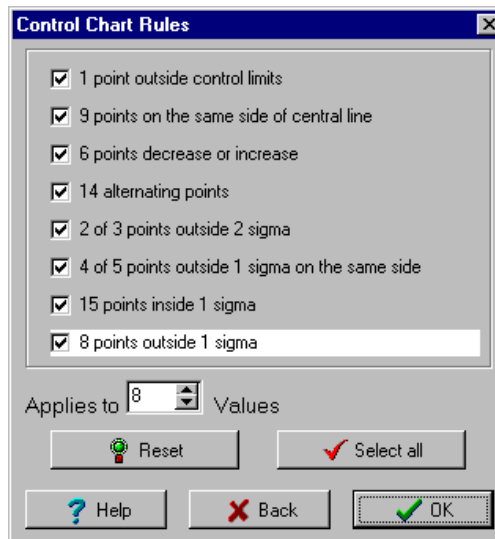


Fig. 5 Rules selection panel

Statistical assumptions

Simple Shewhart control charts X-bar and X-individual should not be used in the following cases:

1. Normality of chart data is rejected, see the Test for normality.
2. Data are dependent or show a linear trend, see Autocorrelation and Test for linear trend.
3. Data are heteroscedastic, i.e. their variance is not constant.
4. Several controlled variables are correlated, see Correlation.

Possible remedies in such situations are:

1. When the departure from normality is caused by an outlier, the outlier should be excluded (only at the chart construction stage). When the departure is caused by systematic skewness or kurtosis of the data distribution, a subgroup size increase might help. Skewness problems can be removed by transformation. High kurtosis problems cannot be simply solved by the transformation technique implemented in the program, so that different quantile construction than the simple 3sigma approach should be used.
2. Detrended data (see data smoothing notes) should be used. The EWMA dynamical chart can be even better.
3. The EWMA dynamical chart should be used.
4. The Hotelling chart for multivariate data should be used.

Capability analysis, capability indices

Process capability indexes (PCI) are used to assess how successful the process control is. The simplest interpretation of a capability index is the following: when it is smaller than 1, process is not capable (does not satisfy given requirements), when the index is larger than 1, process is capable (satisfies given requirements). A more detailed process classification is sometimes used sometimes: capable ($PCI < 1$), not capable ($PCI \geq 1$) and highly capable ($PCI \geq 1.3$). It is more appropriate to use not only the capability index estimates, but to accompany them by their confidence intervals, see *Capability indexes*. A more strict classification then defines capable process as a process with **lower confidence limit for the capability index** larger than 1. The basic C_p index should be used only when the central line is given by the data mean. Confidence interval can be made narrower when more data points are used.

Transformations in control charts

The transformation technique can be helpful when dealing with asymmetrical data distribution. Skewed distributions can occur quite frequently, e.g. for low (trace) pollutant concentrations, product purity level, relative values close to 100%, physical variables like strength, time measurements, volume or surface related variables like mass, size of small particles etc. When the problem is not properly recognized and skewness is not taken into account when constructing a control chart, the chart information value might be drastically reduced. Neglecting skewness during chart construction causes that even if the process is under control, one control limit is frequently exceeded, while the other limit is not reached at all. When the subgroup size is large, effective skewness suppression can be expected, considering the central limit theorem. When significant asymmetry is found in the Transformation module (it is often accompanied by normality rejection in the Basic data analysis module), classical Shewhart chart should not be used. Transformation produces asymmetrical control limits for \bar{X} . Variability chart is based on transformed data, so that the scale of S or R charts differs from the scale of the original data. Capability indexes C_p , C_{pk} , C_{pm} , C_{pmk} are computed from transformed data, which satisfy normality assumption. **Warning:** transformation parameter value has to be the same for both chart construction and chart application!

X-bar and S, X-bar and R charts

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Data and parameters

Data rows correspond to subgroups. Each subgroup has to have at least two values. Missing values are allowed.

Table 1 X-bar chart data, number of subgroups = 10, subgroup size = 3

Time	NO _x 1	NO _x 2	NO _x 3
1.11.96	110.9	101.6	114.6
2.11.96	113.3	120.8	116.3
3.11.96		106.1	107.6
4.11.96	236.2	230.8	238.5
5.11.96	110.3	105.6	104.9
6.11.96	115.6	113.6	122.2
7.11.96	128.3	138.6	127
8.11.96	139.9	133.9	
9.11.96	124.3	127.8	113.9
10.11.96	118.6	118	116.5

Chart construction. Parameters are inputted in the Shewhart control chart construction dialog panel. *X-bar and S* or *X-bar and R* types can be selected in the *Chart type* window. Upon clicking the *Select columns* button, data columns are specified. At least two columns have to be selected. Data should come from a process under statistical control. Occasional outliers or otherwise suspect data as well as data violating specified rules should be omitted. When a transformation is necessary, the transformation parameter can be specified. When the value is not known in advance, it can be computed upon clicking the question mark, ? button. When some of the chart parameters are known in advance, they can be inputted manually upon checking *Manual entry*. The *Rules* button invokes the Rules dialog panel, where some of the rules can be excluded/included by the >>, << buttons. Any change of the rules should be properly justified. Default rules are set by clicking the *Initialize* button. Computed chart parameters can be saved to a file by clicking the *Save parameters* button.

Chart application. As soon as the chart parameters are determined, the chart can be used to control data from the same process. Parameters for control chart application can be set in the Shewhart control chart application dialog panel. The panel is similar to the construction panel, the UCL, LCL limits for *x-bar* and variability chart (S or R) are supposed to be known at this stage. Appropriate data columns are selected in the panel. Chart parameters are either entered manually when the *Manual entry* selection is checked, or they are read from a file upon clicking the *Read parameters* button. The file has to contain parameters for the same chart type and the same subgroup size. Rules can be modified by clicking the Rules button. The *Chart* button produces control chart and protocol output. Chart parameters can be saved by clicking the *Save parameters* button.

Protocol

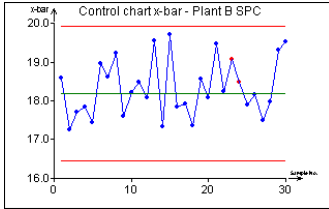
transformation/Transformation	No	Indicates whether any transformation was used to bring data closer to normality. When transformation was applied, control limits can be asymmetrical, depending on data distribution. S chart is based on the transformed data, so that the scale differs from the original data scale.
Chart type	X-bar.	
Maximum subgroup size		Maximum subgroup size, i.e. number of columns, selected upon clicking the Select columns button.
Row number		Number of subgroups, i.e. points plotted in the chart.
Central line		Central line of the chart.
UCL		Upper control limit.
LCL		Lower control limit.
Variability		Specifies how the process variability is expressed.
Baseline		Baseline for standard deviation control chart.
Capability indexes		
Cp		Capability index $C_p = (UCL - LCL) / (6s)$ and its 95% confidence interval.
Cpk		Capability index $C_{pk} = \min(UCL - \text{average}; \text{average} - LCL) / (3s)$ and its 95% confidence interval.
CI (Heavlin)		Heavlin 95% confidence interval.
CI (Kushler)		Kushler 95% confidence interval.
CI (Franklin)		Franklin 95% confidence interval.
Cpm		Capability index $C_{pm} = (UCL - LCL) / (6\sqrt{s^2 + (\text{average} - ZL)^2})$ and its 95% confidence interval.
Cpmk		Combined capability index C_{pmk} .
Rules violations		List of points, violating rules specified previously in the Rules panel.

Points violating rules (corresponding to averages or standard deviations) are printed red bold, violated rule is specified.

Data Printout of the input data.

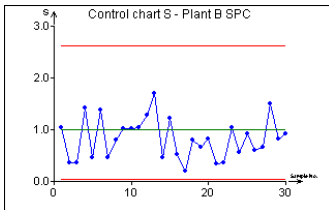
Graphs

X-bar chart



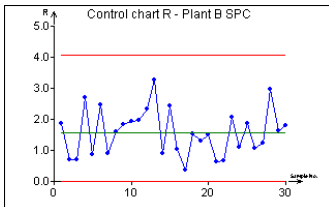
Standard X-bar control chart. Points violating specified rules are marked red. A variability chart (S or R) should be inspected before looking at the X-bar chart. The central line corresponds to the process baseline, the upper and lower lines correspond to the control limits.

S chart



Standard deviation control chart. Points exceeding limits for standard deviation are marked red. Variability charts (S or R) should be inspected before inspecting X-bar or X-individual charts. The green line corresponds to the baseline, the upper and lower lines correspond to the control limits.

R chart



Range control chart. Range is calculated as the maximum-minimum difference within a subgroup. When the subgroup size is larger than 2, S chart is generally preferred. Points exceeding limits for range are marked red. Variability charts (S or R) should be inspected before inspecting X-bar or X-individual charts. The green line corresponds to the baseline, the upper and lower lines correspond to the control limits.

X-individual and R charts

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X-individual chart is used when no rational subgroups can be created. Individual data points are plotted in the chart, hence the chart is much more sensitive to departures from normality than the X-bar charts. X-bar charts should be used whenever possible. Absolute differences of consecutive values are plotted in the R chart.

Data and parameters

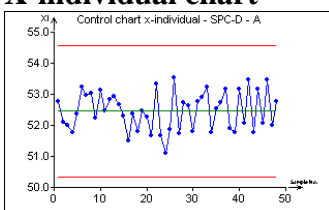
Data are expected in one column. No missing values are allowed. X-individual and R chart type is selected in Shewhart control chart application..

Protocol

X-individual chart protocol is analogous to the X-bar chart protocol.

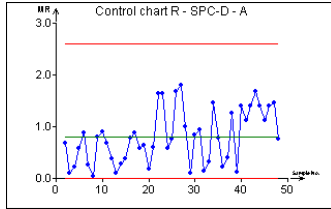
Graphs

X-individual chart



Standard X-individual control chart. Points violating specified rules are marked red. R chart should be inspected before looking at the X-individual chart. The central line corresponds to the process baseline, the upper and lower lines correspond to the control limits.

R chart



Moving range chart. Since each point corresponds to a difference of two consecutive X-individual values, the chart starts from the second time point. The points exceeding limits for standard deviation are marked red. An R chart should be inspected before inspecting the X-individual charts. The green line corresponds to the baseline, the upper and lower lines correspond to the control limits.

np-chart

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The np chart is useful when controlling number of nonconforming items within a batch. A binomial distribution is assumed for the nonconforming items number. It is a chart for integer valued variables, where batch corresponds to subgroup. Number of nonconforming items out of np items contained in a batch is plotted in the chart. Batch size might vary. For discrete quality attributes (like the number of nonconforming items here) no special variability chart is constructed.

Data and parameters

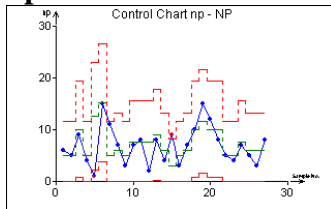
Data are organized in two columns. The number of nonconforming items is in the first column, batch size (i.e. the total number of items in the batch) has to be in the second column. In the *Shewhart control chart construction* dialog panel, the np chart type is selected. Upon clicking the *Select columns* button, data columns are selected. Neither transformation nor additional rules are used for attributes charts (np, p, c, u). When baseline or limit values are known from previous analysis or given by a standard, they can be specified manually in the *Value* window within the Manual entry part of the panel. When the values are not known, they can be computed by clicking the *Calculate* button (*Manual entry* must not be checked). Baseline, LCL, UCL values can be saved into a file by the *Save parameters* button. Pressing the OK button plots the chart.

Protocol

NP	Number of nonconforming items by batches.
N0	Batch sizes.
BL	Baseline for a given batch size.
LCL	Lower control limit for a given batch size.
UCL	Upper control limit for a given batch size.
P	Average proportion of nonconforming items.
Out of limits	List of values out of control limits.

Graphs

np chart



Control chart for the number of nonconforming items. Points falling outside the control limits are marked red. The baseline is blue, the control limits red, their values increase with increasing batch size, hence they are not constant when the batch size changes.

p chart

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The p chart is useful when controlling proportion of nonconforming items in a batch. Because the number of nonconforming items follows a binomial distribution, it is a chart for attributes. Batch corresponds to subgroup. The proportion of nonconforming items within a batch is what is plotted in

the chart. Therefore, the chart value is always between zero and one. The batch size might vary. No separate chart for variability is constructed.

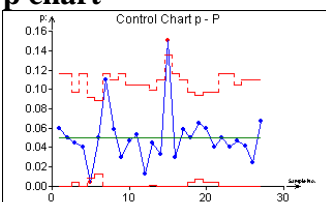
Data and parameters

Data are organized in two columns. Proportion of nonconforming items is in the first column, batch size (i.e. the total number of items in the batch) has to be in the second column. In the *Shewhart control chart construction* dialog panel, the p chart type is selected. Upon clicking the *Select columns* button, data columns are selected. Neither transformation nor additional rules are used for attributes charts (np, p, c, u). When baseline or limit values are known from a previous analysis or given by a standard, they can be specified manually in the *Value* window within the *Manual entry* part of the panel. When the values are not known, they can be computed by clicking the *Calculate* button (*Manual entry* must not be checked). Baseline, LCL, UCL values can be saved into a file by *Save parameters*. Pressing the OK button plots the chart.

Protocol

P	Proportion of nonconforming items by batches.
N0	Batch sizes.
LCL	Lower control limit for a given batch size.
UCL	Upper control limit for a given batch size.
BL	Baseline.
Out of limits	List of points falling out of control limits.

Graphs

<p>p chart</p> 	<p>Control chart for proportion of nonconforming items. Points falling outside the control limits are marked red. The baseline is blue, the control limits are red, all of them depend on the batch size. When the batch size changes, their values are not constant but increase with the sample size.</p>
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c chart

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c chart is a less frequently used chart type. It is a chart for controlling the number of faults found in a defined amount of product. Such number can have only a nonnegative integer value and is assumed to follow a Poisson distribution. Because the controlled characteristic is discrete, the chart is a control chart for attributes. Unlike for np or p charts, the number of faults of the same type is recorded on each piece of product (or a small group of related pieces), where each individual fault might or might not cause whole piece to fail (e.g. number of knots counted within 10 meters of a fabric). The number of faults *c* is plotted in the chart and the size of a controlled piece of product has to stay constant. No special chart for variability is constructed.

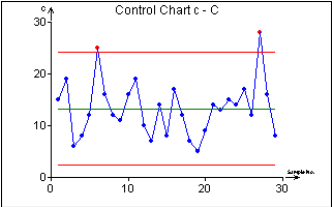
Data and parameters

Data, i.e. the fault counts, counted on individual product pieces are in one column. The c chart is selected in the *Shewhart control chart construction* dialog panel. Data column is specified upon pressing the *Select columns* button. Neither transformation nor special rules are used for any of the attributes control charts (np, p, c, u). When the baseline or the control limits are known from previous data or given by a standard, they can be inputted upon checking *Manual entry*. When their values are not known, *Manual entry* must not be checked and their computation is requested by the *Calculate* button. The *Save parameters* button saves the baseline and the control limits to a file. Pressing the OK button produces the chart.

Protocol

Data	Number of faults counted on individual product pieces.
BL	Baseline.
LCL	Lower control limit.
UCL	Upper control limit.
Out of limits	List of values falling out of control limits.

Graphs

<p>c-chart</p> 	Control chart for controlling number of faults. Data exceeding control limits are marked red. The baseline is green. The control limits are red and constant.
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u chart

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This chart type is utilized less frequently. It is used to control number of faults on a piece of product of variable size. Such number can have only nonnegative integer values and is assumed to follow a Poisson distribution. Because the controlled characteristic is discrete, the c chart is a control chart for attributes. Unlike for np or p charts, the number of faults of the same type is recorded on each piece of product (or a small group of related pieces), where each individual fault might or might not cause the whole piece to fail (e.g. number of knots counted within 10 meters of a fabric). The number of faults u is plotted in the chart. The size of a controlled piece of product is allowed to vary (this is the only difference from the c chart). No special chart for variability is constructed.

Data and parameters

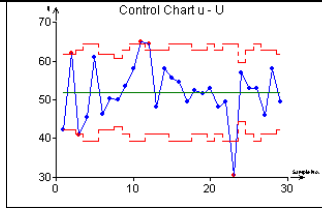
Data, i.e. the fault numbers, counted for individual product pieces are in one column. Sizes of inspected product pieces have to be in the second column. The u chart is selected in the *Shewhart control chart construction* dialog panel. Data column is specified upon pressing the *Select columns* button. Neither transformation nor special rules are used for any of the attributes control charts (np, p, c, u). When the baseline or the control limits are known from previous data or given by a standard, they can be inputted upon checking *Manual entry*. When their values are not known, *Manual entry* must not be checked and their computation is requested by the *Calculate* button. The *Save parameters* button saves the baseline and the control limits to a file. Pressing the *OK* button produces the chart.

Protocol

U	Number of faults counted on inspected pieces of product.
N0	Sizes/amounts of product controlled.
U1	Relative number of faults, standardized to one unit of product, $U1=U/N0$. This is what is plotted in the chart.
LCL	Lower control limit.
UCL	Upper control limit.
BL	Baseline.
Out of control limits	List of points falling outside control limits.

Graphs

u chart	Control chart for fault numbers. Points falling out of control limits are
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marked red. The baseline is green, the control limits are red. The control limits depend on the amount /size of the product controlled. When the amount/size varies, the limits are not constant.