

## Single sample acceptance design

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Single acceptance sampling plan is constructed to decide about a given lot meets a given quality criteria expressed as the actual fraction of non-conforming units in the whole shipment of  $N$  units – the *Acceptable Quality Level (AQL)*. The methodology follows the standard ISO 2859-2. Statistical sampling procedures avoid 100% testing of all items (which may be too costly, or even impossible, e.g. in case of destructive tests). Therefore, only a randomly selected sample is tested for conformity and based on the number of non-conforming units in the sample, we decide about accepting or rejecting the whole shipment. Apparently, this decision may always be correct, or incorrect. The probability (or risk) of incorrect decision is a part of the acceptance sampling parameters defined prior to the sampling procedure by an agreement between supplier and consumer. Two kinds of risk can be defined:

- risk (probability)  $\alpha$  of incorrect rejecting the shipment in which the actual number of non-conforming units lower than the agreed quality level and should have been accepted (so called supplier risk).
- risk (probability)  $\beta$  of incorrect accepting the shipment in which the actual number of non-conforming units is in fact higher than the agreed quality level and should have been rejected instead (so called consumer risk).

It is indeed desirable for both  $\alpha$  and  $\beta$  to be small. For a 100% testing both  $\alpha$  and  $\beta$  would be zero. The sampling agreement seeks a compromise between risks ( $\alpha$  and  $\beta$ ) and testing cost which is a (perhaps linear) function of the number of tested units  $n$ , preferably  $n \ll N$ .

Before performing the acceptance sampling, the supplier and the consumer need to agree on the following:

**Acceptable quality level AQL**, expressed in terms of the actual fraction of non-conforming units which is acceptable and expected for the consumer at the given price, technology level, etc. AQL can be generally understood as the central line in the  $p$ -control chart. The probability of accepting a shipment with the actual fraction of non-conforming equal to AQL should be  $(1 - \alpha)$ .

**Supplier risk  $\alpha$** , the probability of rejecting a shipment in which the fraction of non-conforming units is exactly equal to AQL;

**Minimal unacceptable (rejectable) quality level RQL**, or the actual fraction of non-conforming units which is just unacceptable for the consumer at the given price, technology level, etc. If the actual fraction of non-conforming units is equal to RQL, the consumer is willing to reject this shipment with probability  $(1 - \beta)$ . It is obvious that  $AQL < RQL$ . Smaller difference between AQL and RQL generally requires greater sample size  $n$ , see examples on Fig. 3 and Fig. 4.

**Consumer risk  $\beta$** , the probability of accepting a shipment in which the fraction of non-conforming units is equal to RQL.

*Some examples of the agreed acceptance parameters can be:*

AQL = 0.04; $\alpha$ = 0.05; RQL = 0.1; $\beta$ = 0.05
AQL = 0.005; $\alpha$ = 0.03; RQL = 0.02; $\beta$ = 0.1
AQL = 0.05; $\alpha$ = 0.02; RQL = 0.20; $\beta$ = 0.02

Further quantity that enter the acceptance procedure is the size of the shipment (or a lot, box, etc.). Based on the five quantities (AQL, RQL,  $\alpha$ ,  $\beta$ ,  $N$ ) we must determine the required number  $n$  of

units to be inspected (the sample size) and the maximal number  $c$  of non-conforming units in this sample with which we decide to accept the shipment (so called acceptance number). The three values  $(N, n, c)$  are called the acceptance sampling plan.

*To summarize the acceptance procedure:*

- (1) (preposition) The whole shipment of  $N$  units contains an unknown number  $D$  ( $0 \leq D \leq N$ ) non-conforming units.
- (2) Using this module, determine  $n$  and  $c$  from AQL, RQL,  $\alpha$ ,  $\beta$ ,  $N$ .
- (3) From this whole shipment of  $N$  units select randomly a sample of size  $n$ .
- (4) Inspect all  $n$  selected units for conformity. Denote  $d$  the number of non-conforming units found ( $d \leq n$ ).
- (5) If  $d$  is greater than  $c$  then accept the shipment, otherwise the whole shipment is rejected.

***It is crucial to pay attention to the randomness of the random sampling procedure***

Sampling must be representative, uniform and random (any unit must have the same probability  $1/N$  to be selected into the sample). Reliable uniform random number generators should be used to select numbered items.

An important quantity in sampling is the probability of acceptance  $P_a$  at a given level of quality  $p=D/N$  and a given  $c$  and  $n$ . Actual (unknown) quality is called quality level, QL. Dependence of  $P_a$  on QL is called operation characteristic curve, or OC curve.

$$P_a = P(x \leq c) = \sum_{x=0}^c P(x)$$

where  $P(x)$  is the probability of finding  $x$  non-conforming units in the sample, which has a hypergeometric distribution.

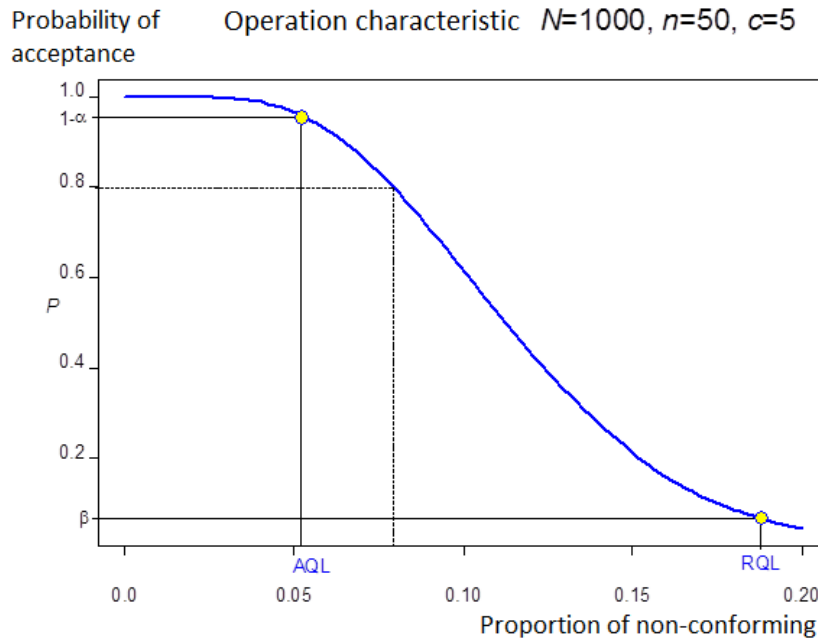
$$P(x) = \frac{\binom{D}{x} \binom{N-D}{n-x}}{\binom{N}{n}}$$

***Operation characteristic curve***

This curve gives the probability  $P_a$  of accepting a lot given  $n$  and  $c$  as a function of actual quality level. The unknown values  $n$  and  $c$  are computed from the set of equations:

$1 - \alpha = \sum_{x=0}^c \frac{n^x (AQL)^x e^{-n(AQL)}}{x!}$ $\beta = \sum_{x=0}^c \frac{n^x (RQL)^x e^{-n(RQL)}}{x!}$	<b>(1-1)</b>
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The resulting real value of  $n$  is then rounded to be the closest higher integer. These equations can not be solved analytically, a numerical iteration algorithm must be employed.



**Fig. 1 An example of an OC curve (probability of acceptance as a function of quality level QL)**

Fig. 1 illustrates a typical OC curve with a visualized sampling plan for a given ( $N=1000$ , AQL, RQL,  $\alpha$ ,  $\beta$ ). If, for example, the actual QL of the lot was 0.08 (8%), the probability of accepting will be 80% (the dotted line). Computed values  $n=50$  a  $c=5$ . Because  $n$  must be integer, there exists no OC curve that meets exactly the requirements (AQL, RQL,  $\alpha$ ,  $\beta$ ). Therefore the real resulting risk must be recalculated for the corrected (rounded)  $n$  to give slightly different actual values  $\alpha'$  and  $\beta'$ . The solution for  $c$  and  $n$  is taken such that  $\alpha' \leq \alpha$  a  $\beta' \leq \beta$  to ensure that the risk requirement is securely met. The AQL and RQL points are visible on the OC curve on Fig. 1.

## Data and parameters

This module requires no data. All parameters *AQL*, *RQL*, *alpha*, *beta* and the lot size *N* are entered in the dialog window, Fig. 2. Pressing Calculate design will provide sample size *n*, acceptance number *c*, rejectance number *c+1*, recalculated actual risk  $\alpha'$  and  $\beta'$  for the integer *n* and *c* and the probability of acceptance  $1 - \alpha$  at the quality level AQL. Clicking on *Plot OC curve* will plot the operation characteristic curve of the calculated design. Clicking on *Protocol* will generate output table for the calculated design in the *Protocol*.

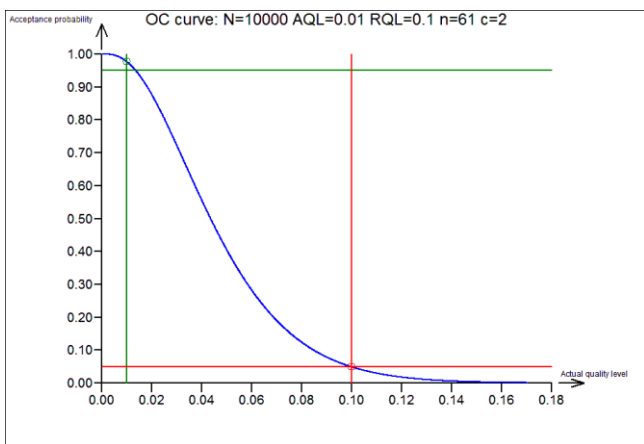
**Fig. 2 Dialog window, Acceptance sampling**

**Acceptance sampling for isolated lot**  
**Exact design, closest safe solution**  
**based on hypergeometric distribution**

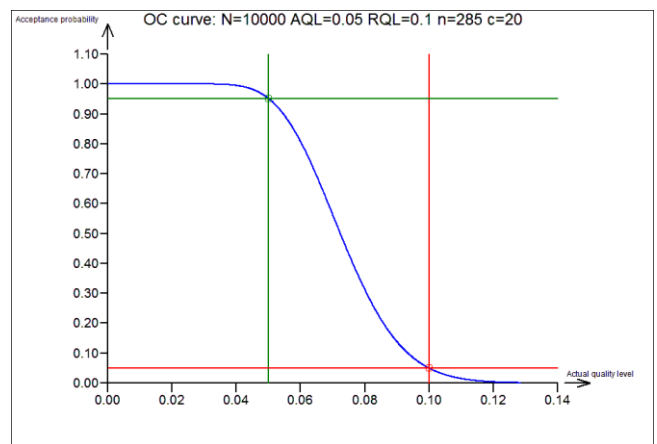
<b>Task name :</b>	Accept_1
Lot size N	10000
Acceptable quality AQL	0.01
Rejectable quality RQL	0.1
Supplier risk alfa	0.05
Consumer risk beta	0.05
<b>Acceptance design:</b>	
Sample size n	61
Acceptance number c	2
Rejectance number c+1	3
Actual supplier risk alfa	0.02299
Actual acceptance probability @ AQL	0.97702
Actual consumer risk beta	0.04864

**Acceptance sampling for isolated lot**  
**Exact design, closest safe solution**  
**based on hypergeometric distribution**

<b>Task name :</b>	Accept_2
Lot size N	10000
Acceptable quality AQL	0.05
Rejectable quality RQL	0.1
Supplier risk alfa	0.05
Consumer risk beta	0.05
<b>Acceptance design:</b>	
Sample size n	285
Acceptance number c	20
Rejectance number c+1	21
Actual supplier risk alfa	0.04831
Actual acceptance probability @ AQL	0.95169
Actual consumer risk beta	0.04952



**Fig. 3 Text and graphical output for N=10000, AQL=0.01, RQL=0.1, calculated design: n=61, c=2**



**Fig. 4 Text and graphical output for N=10000, AQL=0.05, RQL=0.1; Small difference between AQL and RQL leads to considerably larger samples, n=285**

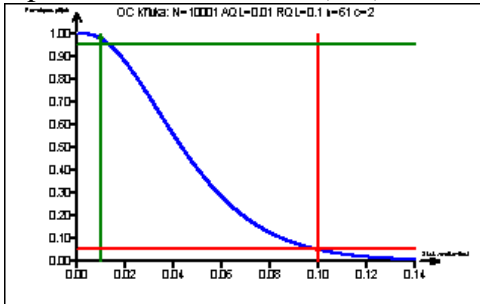
## Protocol

Lot size N	Number of units in the whole amount to be accepted (lot, shipment, containment, box, etc.) .
Acceptable quality AQL	Acceptable quality level as fraction of non-conforming.
Rejectable quality RQL	Rejectable quality level as fraction of non-conforming.
Supplier risk alpha	Required probability of rejecting a lot at quality level equal to AQL.
Consumer risk beta	Required probability of accepting a lot at quality level equal to RQL.
Acceptance design:	
Sample size n	Calculated sample size $n$ (number of randomly sampled units to inspect).
Acceptance number c	Maximal acceptable number of non-conforming in the sample.
Rejectance number c+1	Lowest unacceptable number of non-conforming in the sample.

Actual supplier risk alpha	Actually reached supplier risk $\alpha'$ at the design $(n, c)$
Actual acceptance probability @ AQL	Actual probability $1 - \alpha'$ of accepting a lot at AQL by this plan $(n, c)$ .
Actual consumer risk beta	Actually reached consumer risk $\beta'$ at the design $(n, c)$

## Graphs

### Operation characteristic (OC) curve



OC curve is a dependence of probability of acceptance a lot a given  $n$  and  $c$  on the actual fraction of non-conforming in the lot. The rejectable quality level (RQL) and the corresponding actual probability  $\beta'$  of accepting at RQL, is drawn as red vertical and horizontal line. The acceptable quality level (AQL) and the corresponding actual probability  $(1 - \alpha')$  of accepting at AQL, is drawn as green vertical and horizontal line. The OC curve represents the closest possible sampling design with minimal difference  $(\alpha - \alpha')$  and  $(\beta - \beta')$  with  $\alpha' \leq \alpha$  and  $\beta' \leq \beta$ .